DOCUMENT RESUME

ED 187 595 SE 031 114

TITLE	The Arithmetic Project Course for Teachers - 14. Topic: Two Points to Two Points on a Number Line. Supplement: Graphing Number Line Jumping Rules, Part Two.
INSTITUTION	Education Development Center, Inc., Newton, Mass.; Illinois Univ., Urbana.
SPONS AGENCY	Carnegie Corp. of New York, N.Y.; National Science Foundation, Washington, D.C.
PUB DATE	73
NOTE	23p.; For related documents, see SE 031 100-120. Contains occasional light type.
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Algorithms; Elementary Education; *Elementary School Mathematics; Elementary School Teachers; Inservice Education; *Inservice Teacher Education; Mathematics Curriculum; *Mathematics Instruction; Mathematics Teachers; Number Concepts; *Problem Sets; Teacher Education
IDENTIFIERS	*University of Illinois Arithmetic Project

ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains exercises on moving two points to two points on a number line, a summary of the problems from the film "Competing Number Line Rules," and the supplement. (MK)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED187595

THE ARITHMETIC PROJECT COURSE FOR TEACHERS

U S DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Mr. Murphy of EDC
Mary L. Charles of NSF

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

TOPIC: Two Points to Two Points on a Number Line.

FILM: Competing Number Line Rules, Grade 5

SUPPLEMENT: Graphing Number Line Jumping Rules, Part Two

NAME: .....

14

SE 031 114

This booklet is part of a course for teachers produced by The Arithmetic Project in association with Education Development Center. Principal financial support has come from the Carnegie Corporation of New York, the University of Illinois, and the National Science Foundation.

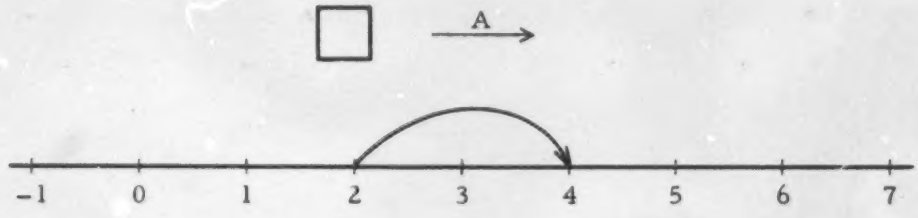
The course is available from:

THE ARITHMETIC PROJECT
Education Development Center
55 Chapel Street
Newton, Massachusetts 02160

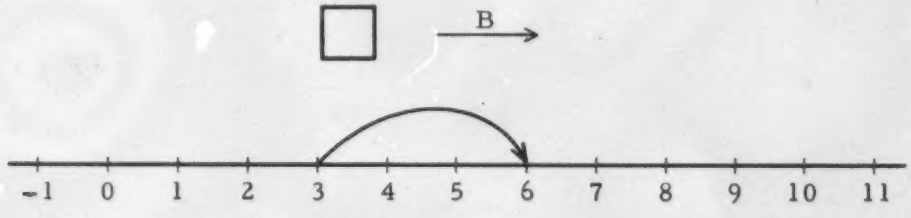
University of Illinois Arithmetic Project
Copyright © 1968, 1973 by Education Development Center, Inc.
All rights reserved.

TWO POINTS TO TWO POINTS ON A NUMBER LINE

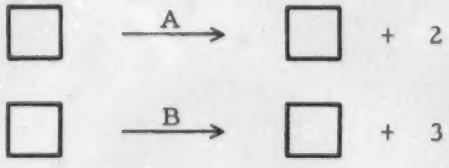
Write a rule which has as one of its jumps the jump from 2 to 4 :



Write a rule which jumps from 3 to 6 :



One way of answering the previous questions is:



A new question (which your earlier choice of rule A and rule B may have answered) is: Can you find one rule which has jumps from 2 to 4 and also from 3 to 6 ? Neither rule A nor rule B given above does this double job. Write a rule that does:

With your rule C you should land on 4 if you start at 2, and you should land on 6 if you start at 3.

The following sequences of problems are intended to show how, in general, to find a rule that takes you from any pair of starting points to any pair of landing points. For example, a problem later on will be to find <u>one</u> rule which takes you from starting points 6 and 13 to landing points $1\frac{1}{2}$ and $43\frac{1}{2}$. (You are not supposed to be able to do this one now! Work the problems in order—preferably at one sitting.)

1. Given the starting points 9 and 19, write one rule that takes you to landing points 10 and 20, as indicated.

START		LAND
9	⟶	10
19	⟶	20

□ ⟶

2. Here is a single rule that does this:

START		LAND
15	⟶	20
45	⟶	50

□ ⟶ □ + 5

3. Write a rule that does this:

START		LAND
15	⟶	23
45	⟶	53

□ ⟶

(Note: If you wrote simply "8", you forgot something. With the rule □ ⟶ 8 you never land at 23 or 53. You go to 8 no matter where you start.)

4. Write a rule:

START		LAND
15	⟶	13
45	⟶	43

□ ⟶

5. Write a rule:

START		LAND
15	⟶	0
45	⟶	30

□ ⟶

6. Write a rule:

START		LAND
15	⟶	-5
45	⟶	25

□ ⟶

7. Write a rule:

START		LAND
16	⟶	100
37	⟶	121

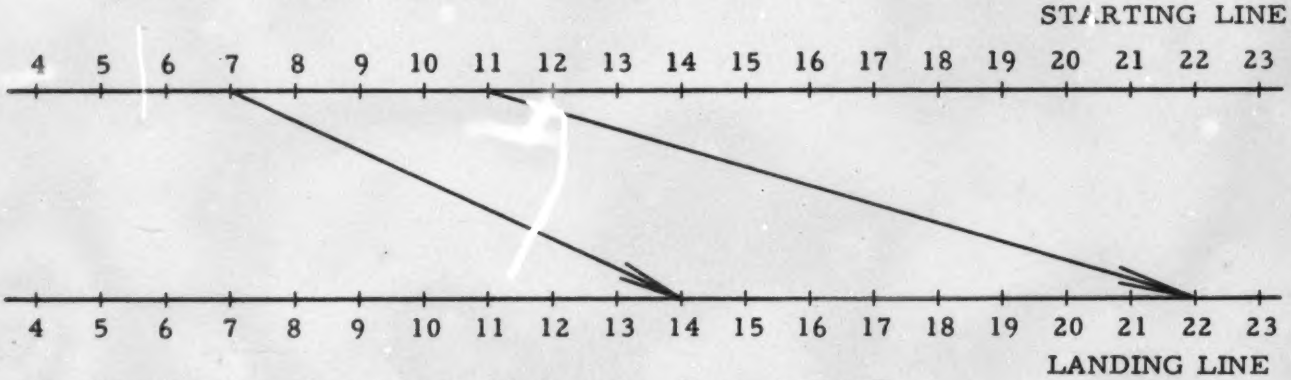
□ ⟶

8. Write a rule:

START		LAND
$\frac{3}{20}$	⟶	$1\frac{1}{60}$
$\frac{7}{15}$	⟶	$1\frac{1}{3}$

□ ⟶

You may have noticed in the previous eight questions that the difference between the two starting points was the same as the difference between their corresponding landing points. In the following illustration this is not the case.



The diagram shows two jumps with the rule □ ⟶ 2 × □ .

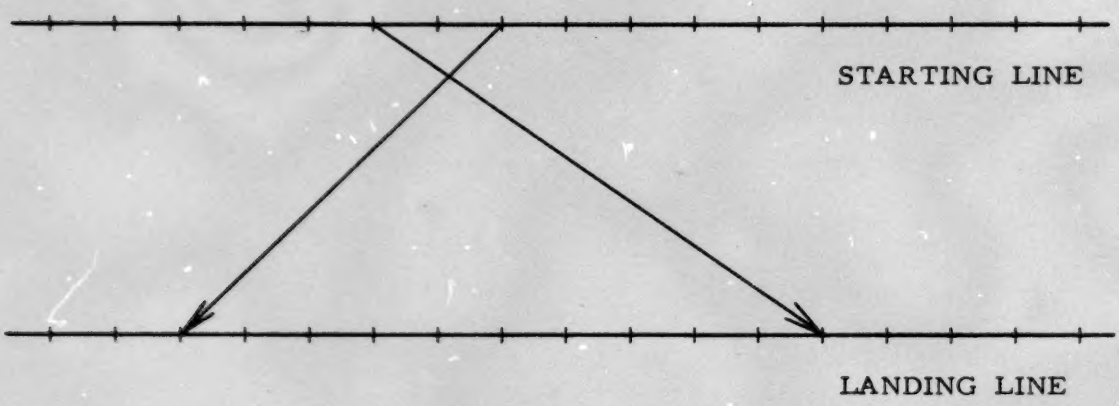
Now complete the following table (the first line is completed as a sample):

	Starting Points	Landing Points	1st Starting pt. minus 2nd Starting pt.	1st Landing pt. minus 2nd Landing pt.	Rule
1.	18 → 9 8 → 4		10	5	□ → $\frac{1}{2}$ × □
2.	11 → 22 7 → 14				□ →
3.	7 → 21 2 → 6				□ →
4.	2 → 6 7 → 21			-15	
5.	28 → ____ 19 → ____				□ → 3 × □
6.	____ → 33 ____ → 51				□ → 3 × □
7.	____ → 33 ____ → 51				□ → $\frac{1}{3}$ × □
8.	10 → ____ 5 → ____		5	20	
9.	____ → ____ ____ → ____		6	12	
10.	____ → ____ ____ → ____		6	18	
11.	____ → ____ ____ → ____		6	15	

☆12. Comment on problems 9 - 11.

	Starting Points	Landing Points	1st Starting pt. minus 2nd Starting pt.	1st Landing pt. minus 2nd Landing pt.	Rule
13.	8 ⟶	-16		-12	
	2 ⟶	-4			
14.	4 ⟶	_____			□ ⟶ -3 × □
	10 ⟶	_____			
15.	16 ⟶	-40		-15	
	10 ⟶	-25			
16.	100 ⟶	_____	200		□ ⟶ 5 × □
	_____ ⟶	_____			
17.	299 ⟶	Leave Blank			□ ⟶ 17 × □
	99 ⟶				
18.	$9\frac{1}{2}$ ⟶	76			
	$4\frac{1}{2}$ ⟶	36			
19.	447,281 ⟶	447,281	Leave Blank	Leave Blank	
	87,009 ⟶	87,009			

☆20. Find a rule such that two of its jumps might be illustrated by the following diagram. You should number your starting points and landing points, and you may calibrate your number lines any way you wish.



Summary of Problems in the Film
"Competing Number Line Rules"

5th Grade, James Russell Lowell School, Watertown, Massachusetts
Teacher: David A. Page

$\square \xrightarrow{a} \square + 20$

Start at 7 and use rule a . (Land: 27)

$\square \xrightarrow{b} 6 \times \square$

Start at 2 and use rule b . (12)

$\square \xrightarrow{c} \square \times \square$

Start at 4, use rule c . Where do you land? (16)

Start at 5, use rule c . Land?

Start at 6, use rule c .

Your starting point is zero. You get to use any two of the rules a , b , or c . Which rules do you pick to get the biggest possible number? Put the letters in $\square$ and $\triangle$:

$0 \xrightarrow{\square\triangle}$ =

Use bc : $0 \xrightarrow{bc}$ = 0

Use ab : $0 \xrightarrow{ab}$ = 120

How many think 120 is the largest number that you can get?

Use ac : $0 \xrightarrow{ac}$ = 400

How can you get the littlest number?

Now the rules are these:

$\square \xrightarrow{a} \square + 7$

$\square \xrightarrow{b} 6 \times \square$

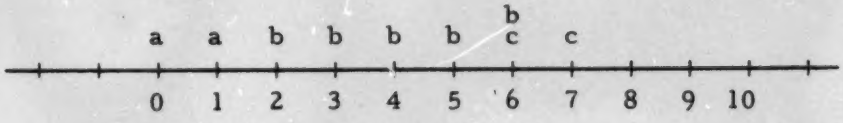
$\square \xrightarrow{c} \square \times \square$

Just using one rule, which rule wins at 0? At 1? At 2? At 3? At 4?

Where is the next interesting number? (6)

What happens at 6? (Answer: b and c both take 6 to 36.)

Which rule wins at 5? At 7?



Which two rules do you pick to get the biggest possible number starting at 1?

Try ac : $1 \xrightarrow{ac}$ = 64

Try bc : $1 \xrightarrow{bc}$ = 36

Which two rules do you pick starting at 2? (Answer: bc)

How come bc wins at 2? (2 in b gives 12 and 2 in a is only 9. "You want to get the biggest possible number before you use c .")

How about 3? How about 4? Will it change as we get to bigger numbers, or will it always be bc ?

Six is an interesting number.

What will you use at 10? (cc)

How about 7? (cc)

Tell me some more about 6. (Answer: 6 could be cc too; it could be either cc or bc .)

New rules:

$\square \xrightarrow{a} \square + 8$

$\square \xrightarrow{b} 4 \times \square$

$\square \xrightarrow{c} \square \times \square$

Make a chart. Start at 0 and figure out which two rules win right up the line, especially at places where things change.

Does anyone know an interesting place?

Why is 4 an interesting place? ("Both b and c work.")

So which ones did you pick at 4? (bc and cc)

Which gave the biggest number at 0, 1, 2? (ac)

Someone says that at 2 you can use ac or bc. Is this correct?

($2 \xrightarrow{ac} = 100$ and

$2 \xrightarrow{bc} = 64$, so ac wins at 2.)

The following chart, offered by a student, is put up on the board:

ac	ac	ac	bc	bc	bc	bc
0	1	2	3	4	5	6

Now start at 4.

($4 \xrightarrow{bc} = 256$ and

$4 \xrightarrow{cc} = 256$, so bc and cc are tied at 4.)

What happens at 5? (From 5 on it's cc .)

New rules:

$\square \xrightarrow{a} \square + 1$

$\square \xrightarrow{b} 2 \times \square$

$\square \xrightarrow{c} \square \times \square$

Tell me the whole story, especially where the interesting numbers are for these rules.

(2 is an interesting number.
It is the same with bc and cc .)

Which would you pick at 3 ? (cc)

At 0 ? (ab)

Tell me something else about 0 . Find a way to tie ab using different letters, starting at 0 .

What about $\frac{1}{2}$? ($\frac{1}{2} \xrightarrow{ba}$ = 2 and

$\frac{1}{2} \xrightarrow{ab}$ = 3 .)

Can you get higher than 3 starting at $\frac{1}{2}$?

Which combination wins at 1 ? ($1 \xrightarrow{bc}$ = 4 and

$1 \xrightarrow{ab}$ = 4 .)

Is there another tie at 1 ? (bb)

How about 2 ? (cc and bc tie.)

What do you want for $2\frac{1}{5}$? (cc)

Why? ($2 \times 2\frac{1}{5}$ is not as much as $2\frac{1}{5} \times 2\frac{1}{5}$.)

What do you want for $1\frac{4}{5}$? (bc)

Why?

A WORD OF WARNING REGARDING TIE POINTS AND SQUARING RULES

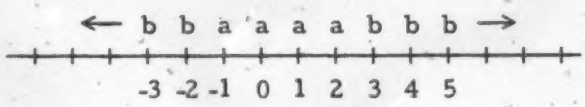
If you are planning to look for tie points with rules involving squaring and adding (or subtracting), be sure to pick your rules carefully and work out the problems ahead of time.

Suppose your rules are:

$$\square \xrightarrow{a} \square + 5$$

$$\square \xrightarrow{b} \square \times \square$$

Choosing either rule a or rule b to get the higher number, you will find that the picture looks like this:



Note that there is a changeover from a to b between 2 and 3, and another changeover between -1 and -2. If you and your class try to hunt down either of the tie points by using rational numbers, you will find that you come quite close. At $2\frac{1}{2}$, for example, rule a gets you to $7\frac{1}{2}$ and rule b gets you to $6\frac{1}{4}$, so rule a wins. At $2\frac{3}{4}$, you will find that a wins with $7\frac{12}{16}$ versus b's $7\frac{9}{16}$. You can keep trying numbers between $2\frac{3}{4}$ and 3 searching for a tie point without avail. Actually the tie points are not rational numbers at all; they turn out to be:

$$\frac{1 + \sqrt{21}}{2} \quad \text{and} \quad \frac{1 - \sqrt{21}}{2}$$

One does not normally expect elementary school students to be familiar with numbers of this kind, although, if they are, so much the better. In any case, the moral is: when you have a rule with $\square \times \square$, you may not always be able to find rational tie points. You can still look for the overall patterns by finding the winning rule for starting points of your choice, as in the film.

Supplement

Graphing Number Line Jumping Rules
Part Two

by Edward Esty

XIV.

The written lesson in this booklet is concerned with finding a rule which would take two specified starting numbers to two specified landing numbers. We can approach this problem in a different way by using the fact that rules of the form $\square \longrightarrow a \times \square + b$ (where a and b are any numbers) have graphs which are straight lines.

1. Suppose we want a rule which will take 1 to 4 and 3 to 10. Draw the dot which shows the jump from 1 to 4. Do the same for the other jump. Now draw the straight line through the two dots. This line is the graph of a rule which takes 1 to 4 and 3 to 10. What is the rule?

$\square \longrightarrow$ _____

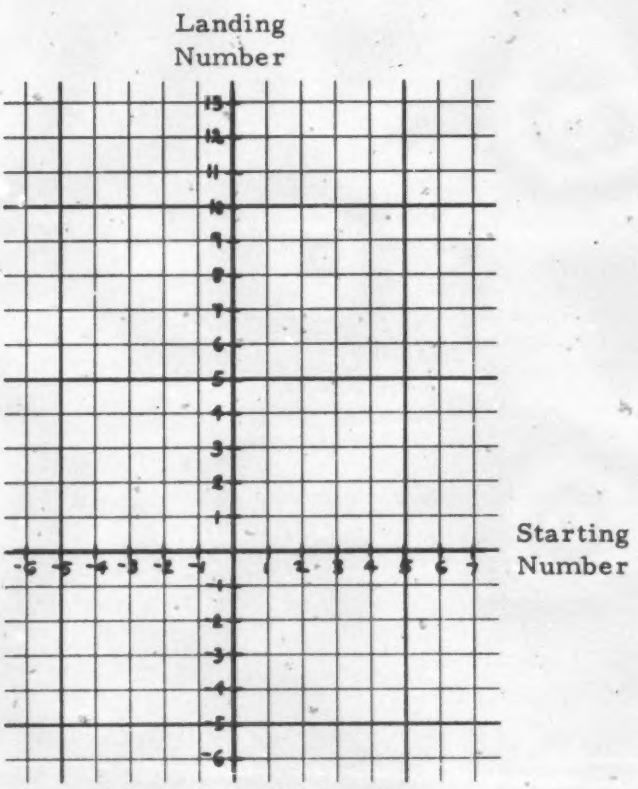


Fig. 25

2. On the graph paper below, use the same procedure to find a rule which sends 6 to -2 and -6 to 1 :

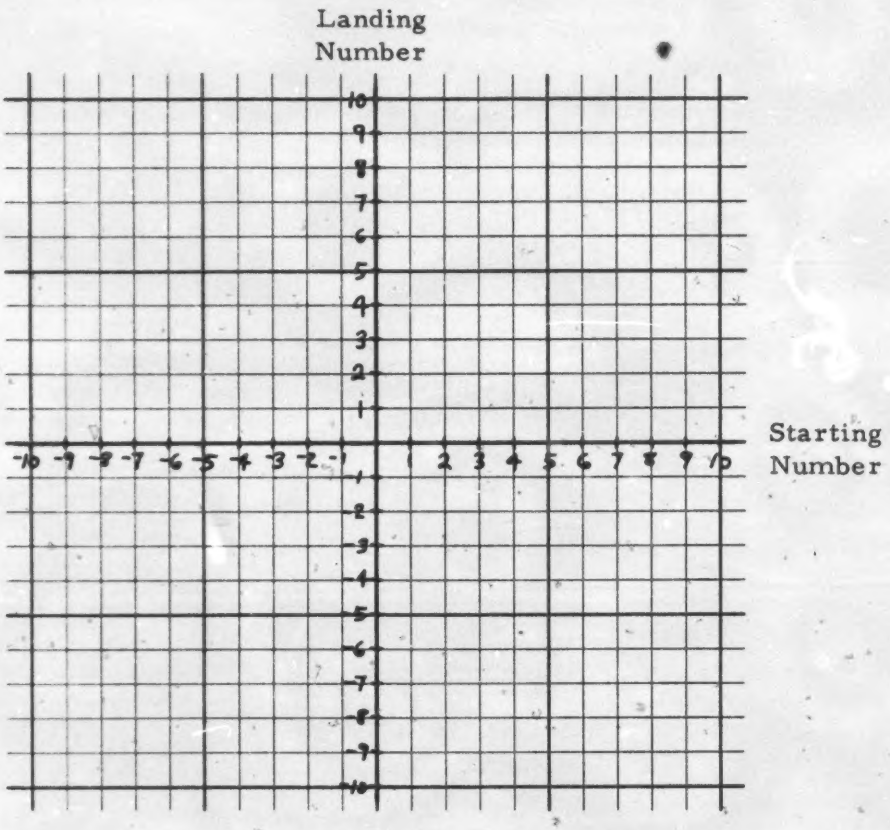


Fig. 26

Rule: □ ⟶ _____

XV.

Here is the graph of the rule $\square \xrightarrow{A} \square \times \square$:

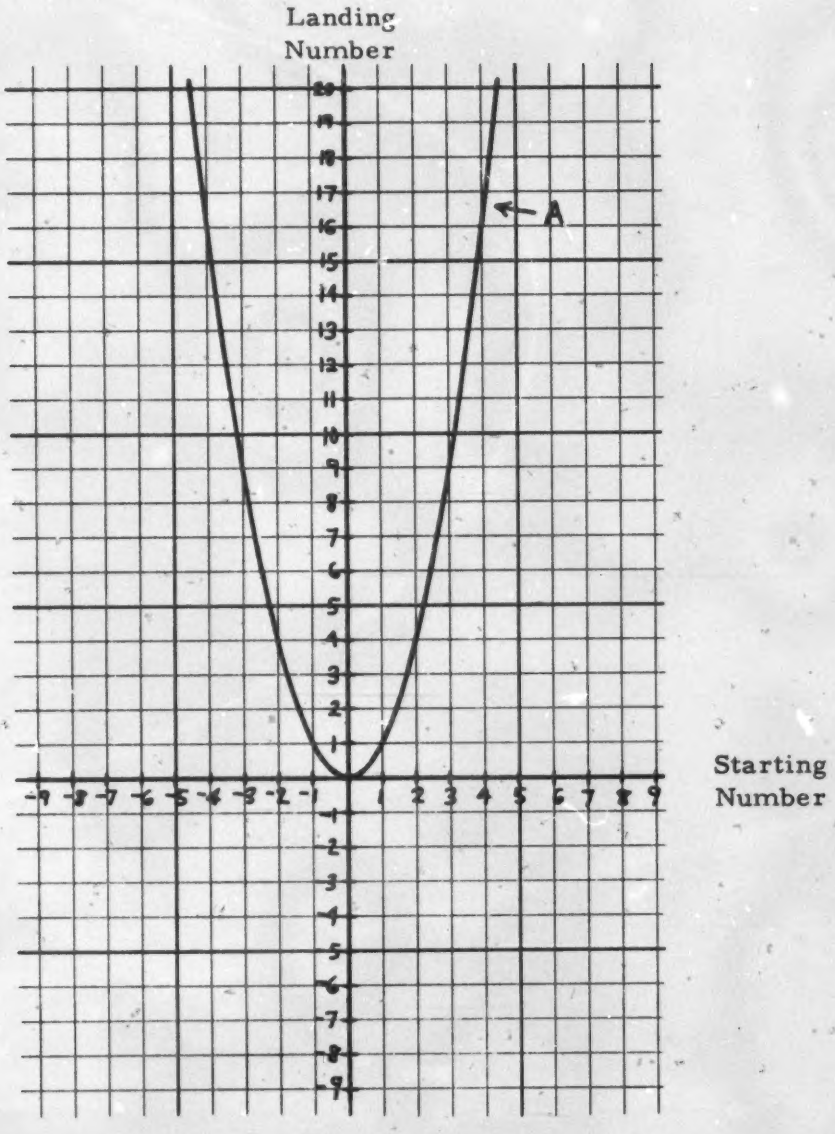


Fig. 27

1. Graph the rule $\square \xrightarrow{B} \frac{1}{4} \times \square \times \square$ on the graph paper above. (Note that the landing numbers for this rule are one fourth the landing numbers given by rule A .)

2. Graph the rule $\square \xrightarrow{C} -\frac{1}{4} \times \square \times \square$. (Note that the landing numbers for this rule are the opposites of the landing numbers given by rule B .)

3. Graph the rule $\square \xrightarrow{D} -\frac{1}{4} \times \square \times \square + 3$.

Write the jumping rules which give these graphs. The first one has been done for you.

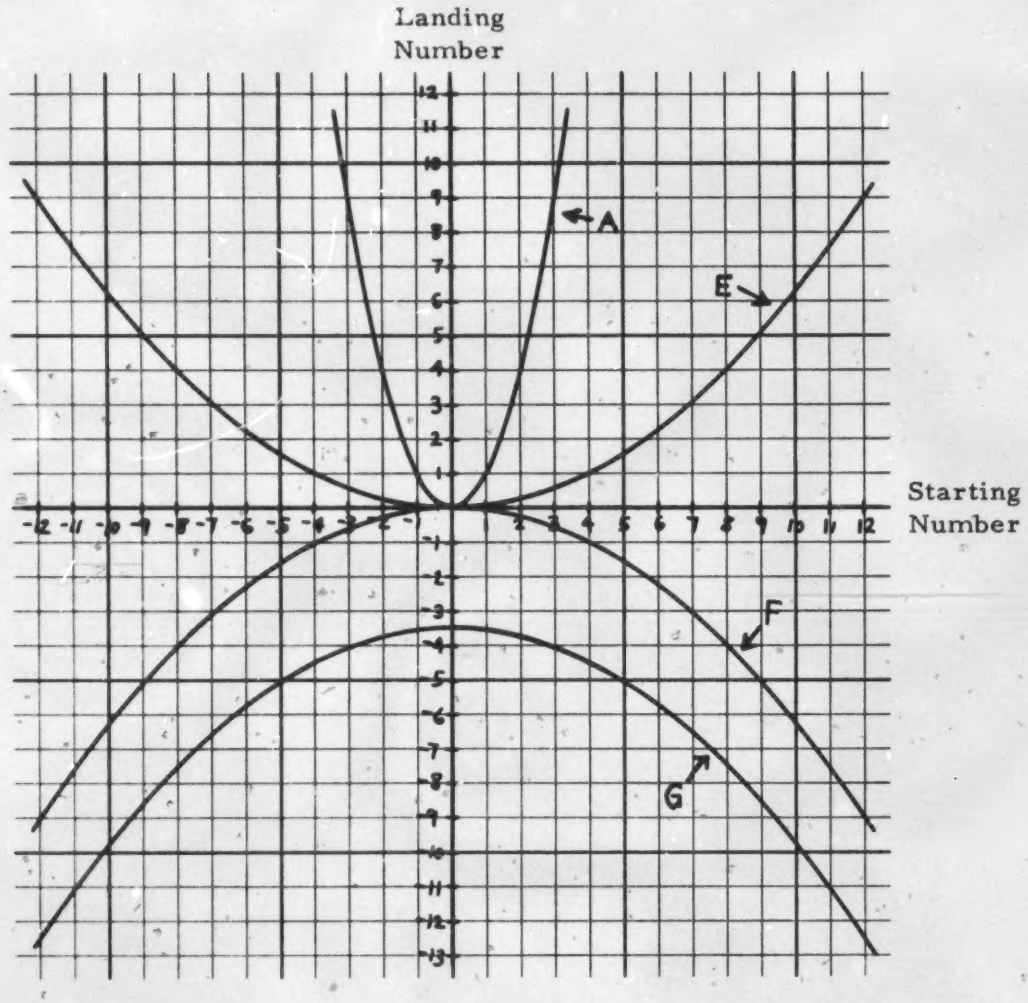


Fig. 28

4. □ $\xrightarrow{A}$ □ × □ _____

5. □ $\xrightarrow{E}$ _____

6. □ $\xrightarrow{F}$ _____

7. □ $\xrightarrow{G}$ _____

Here again is the graph of $\square \xrightarrow{A} \square \times \square$:

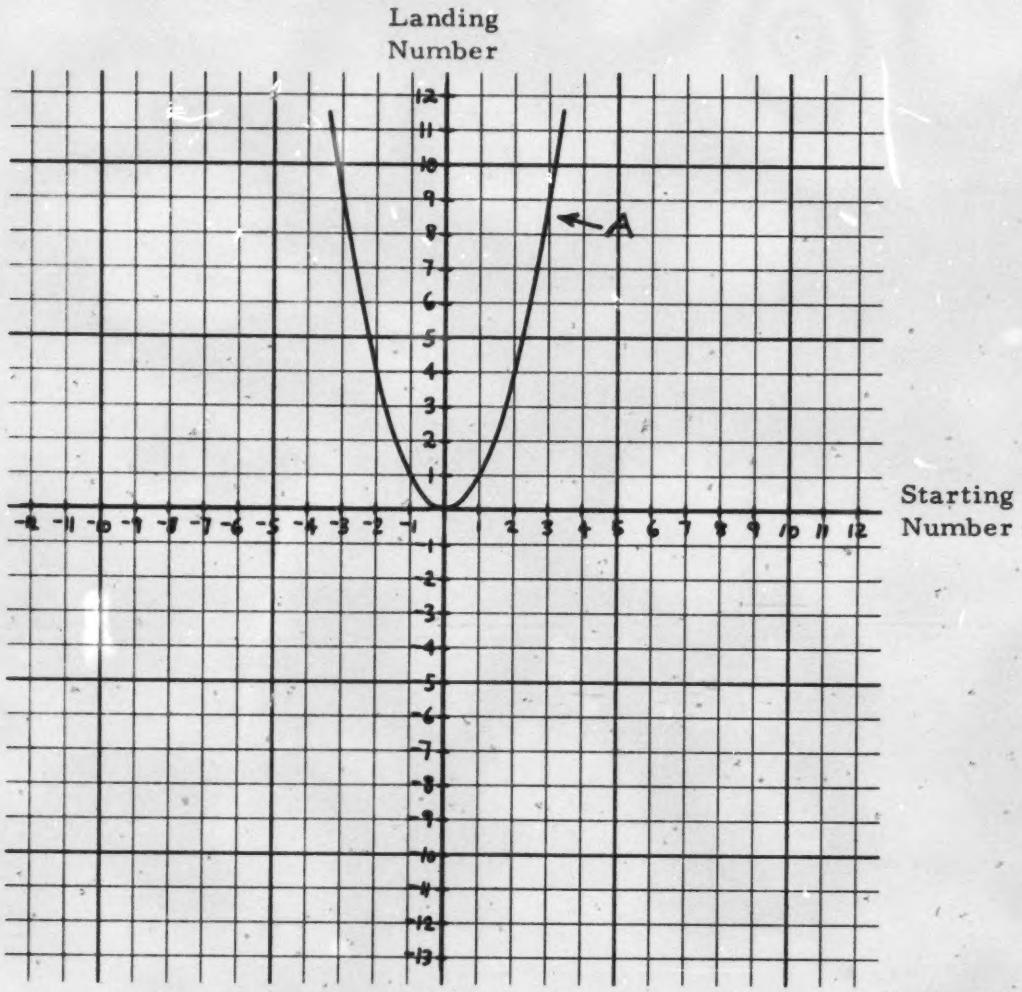


Fig. 29

8. Draw the graph of the rule $\square \xrightarrow{H} (\square - 4) \times (\square - 4)$. Notice that for each starting number, rule H gives the same landing number which rule A gives with a starting number 4 less. That is,

$7 \xrightarrow{H} = 3 \xrightarrow{A}$ and $100 \xrightarrow{H} = 96 \xrightarrow{A}$, etc.

9. Draw the graph of the rule $\square \xrightarrow{J} (\square + 5) \times (\square + 5) - 2$ on Figure 29.

Write the jumping rules which give these graphs:

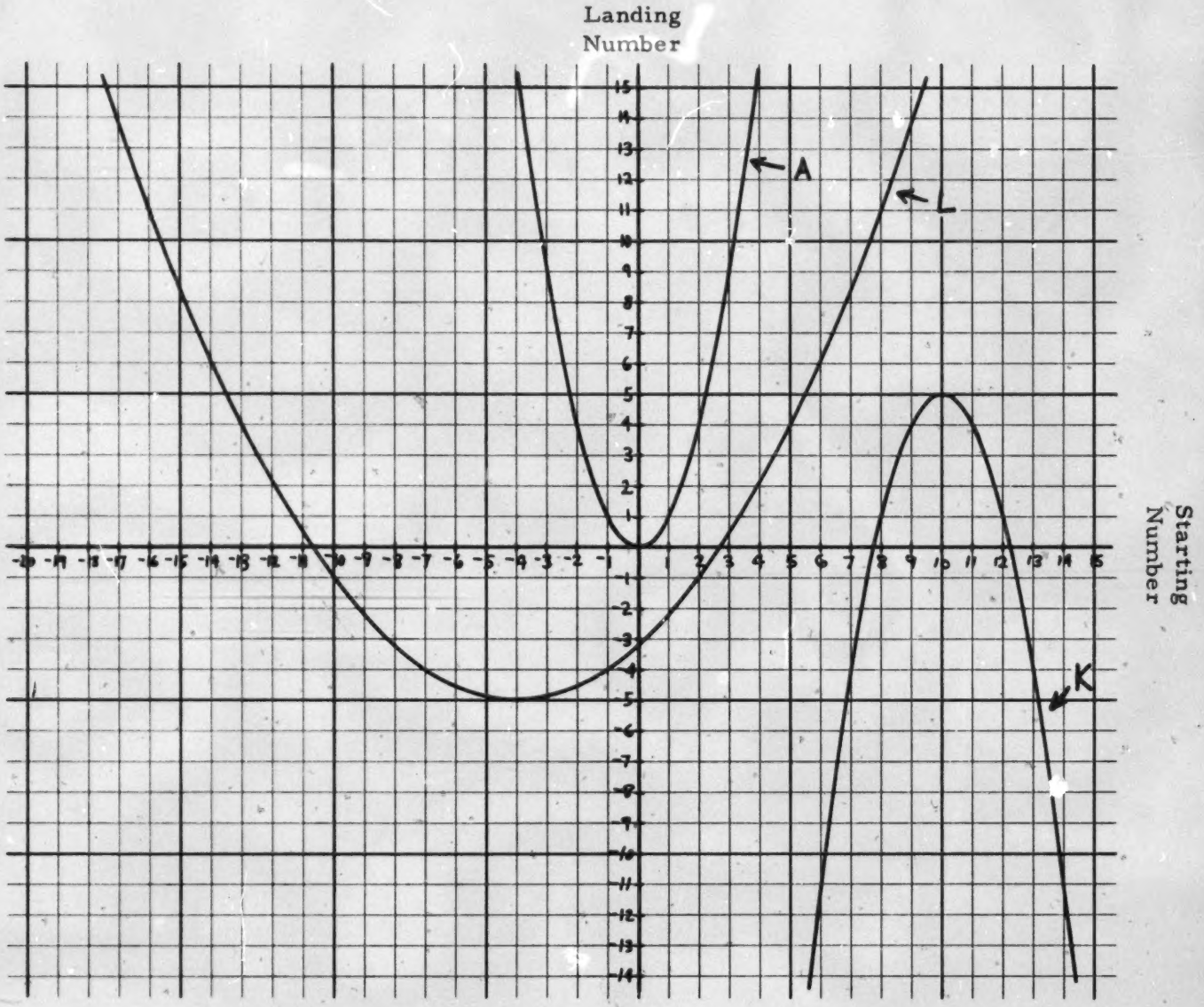


Fig. 30

10. □ $\xrightarrow{A}$ □ × □ _____

11. □ $\xrightarrow{K}$ _____

12. □ $\xrightarrow{L}$ _____

XVI.

The graph of each rule we've worked with so far has been one connected piece. Some graphs were straight lines, others were curved, but none of them has had any sort of break or gap. Here's an example of a rule whose graph has a break in it:

$$\square \longrightarrow \begin{cases} \square + 3 & \text{if } \square \text{ is less than } 4 \\ 2 \times \square - 5 & \text{if } \square \text{ is greater than or equal to } 4 \end{cases}$$

1. Draw the graph of this rule below. Be sure to show clearly the jump that starts at 4.

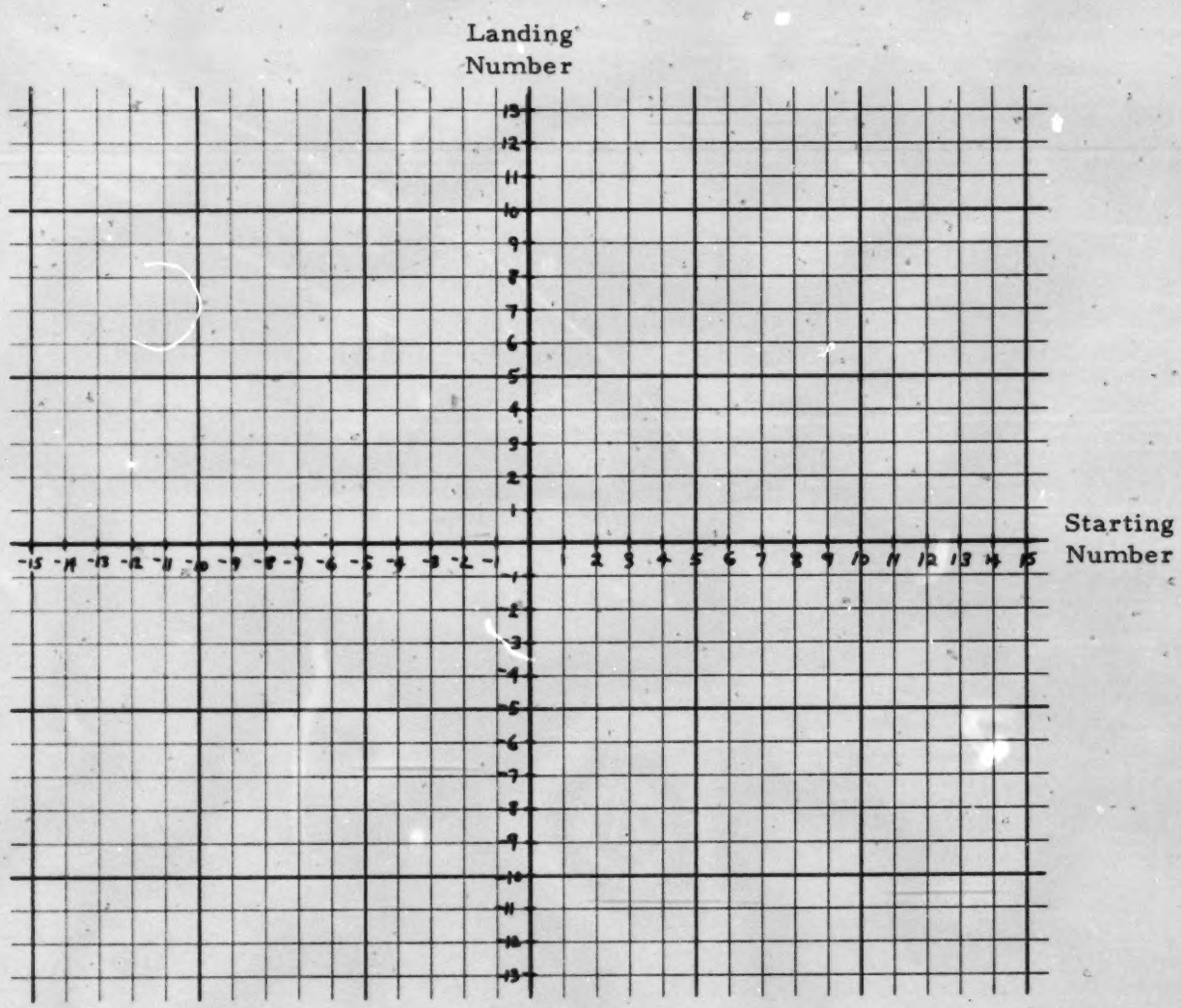


Fig. 31

2. Write the jumping rule whose graph is shown at the right.

□ → _____

(The little parenthesis in the graph shows that the jump starting at 2 lands not at 5, but at 3.)

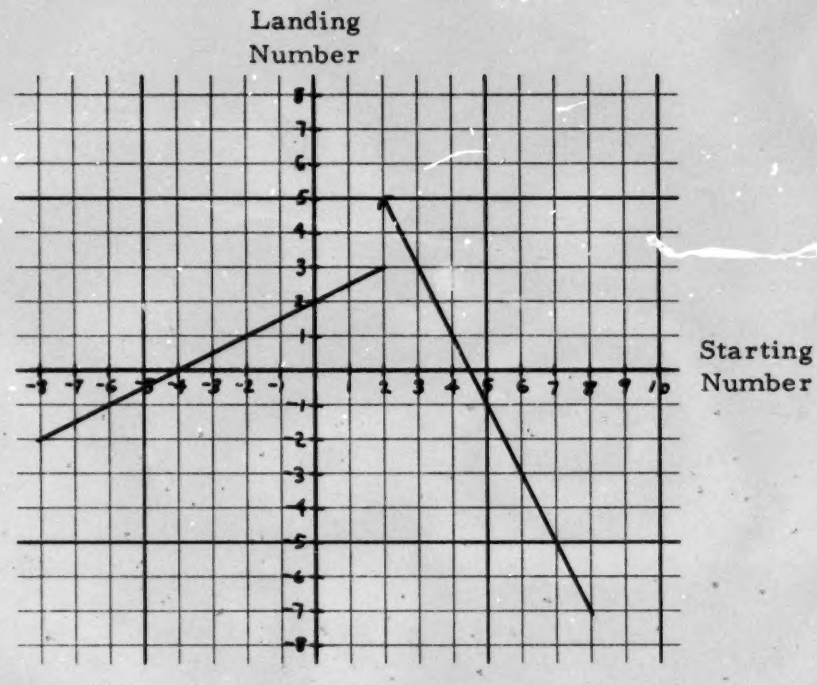


Fig. 32

3. Do the same for this graph.

□ → _____

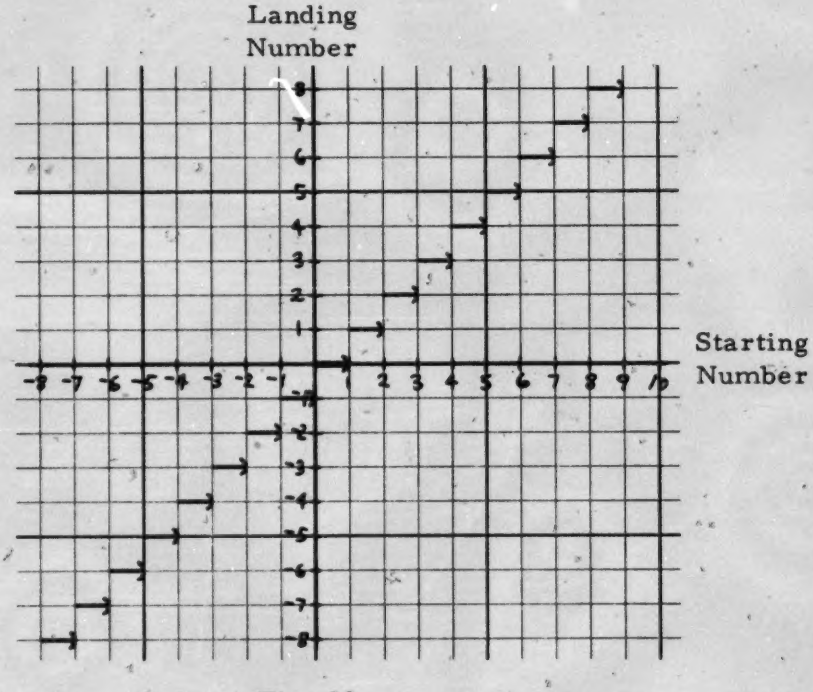


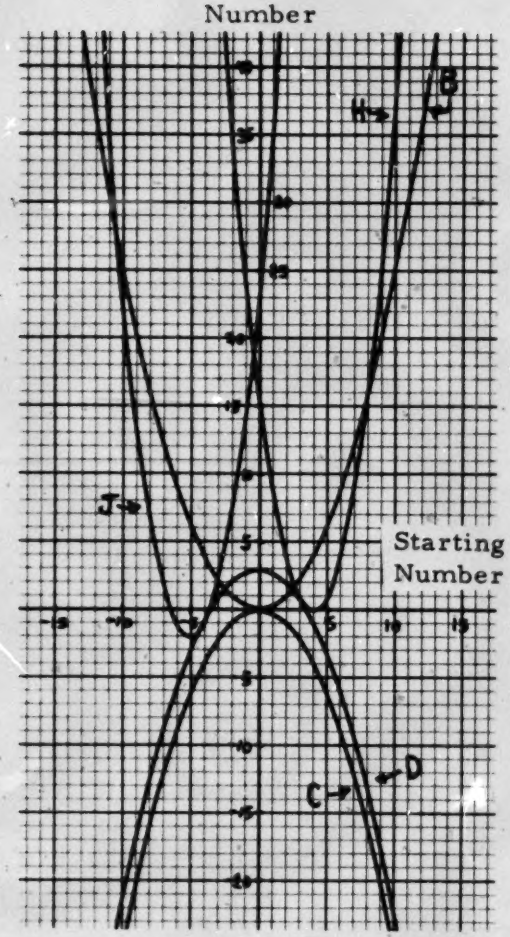
Fig. 33

Answers

XIV. 1. $\square \longrightarrow 3 \times \square + 1$

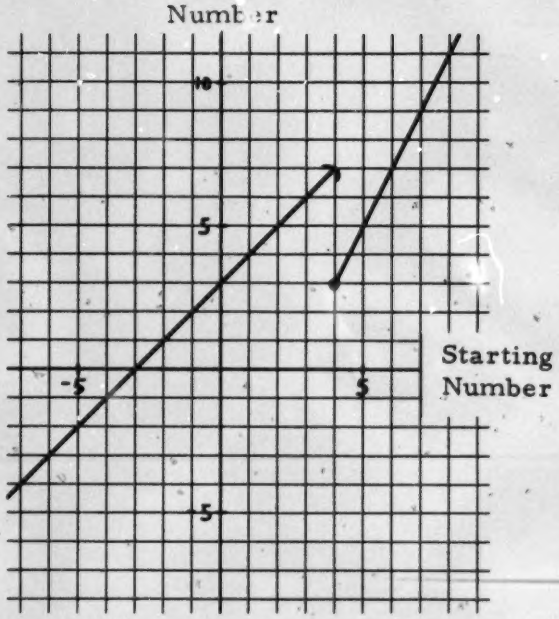
2. $\square \longrightarrow -\frac{1}{4} \times \square - \frac{1}{2}$

XV.



XVI.

1.



2. $\square \longrightarrow \begin{cases} \frac{1}{2} \times \square + 2 & \text{if } \square \leq 2 \\ -2 \times \square + 9 & \text{if } \square > 2 \end{cases}$

3. $\square \longrightarrow \lfloor \square \rfloor$

5. $\square \xrightarrow{E} \frac{1}{16} \times \square \times \square$

6. $\square \xrightarrow{F} -\frac{1}{16} \times \square \times \square$

7. $\square \xrightarrow{G} -\frac{1}{16} \times \square \times \square - 3\frac{1}{2}$

11. $\square \xrightarrow{K} -(\square - 10) \times (\square - 10) + 5$

12. $\square \xrightarrow{L} \frac{1}{9} \times (\square + 4) \times (\square + 4) - 5$